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Mitigating the Effects of Delay Errors in Delay Line Based Sub-Nyquist Frequency Estimation for Military Aerial Vehicle Applications

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***Abstract*—In the delay line based, sub-Nyquist approach to estimate frequencies of radar signals spanning several GHz (say, 2 to 18 GHz), especially for electronic warfare applications, the frequency is estimated from the phase difference between the aliased components of the original signal and a delayed version of the same. This method, however, implicitly assumes the delay employed to be accurately known, though, in practice, no delay (especially analog delay) is free from error that is usually frequency dependent and not known a priori. Since the operating frequency is in the GHz range, even a small error in the delay employed can give rise to significant error in the frequency estimate. To address this issue, this paper first proposes a two delay line based approach where the shorter delay line produces a so-called coarse estimate of the frequency and the longer delay line treats it as a reference to produce more accurate coarse and later fine estimates. Analysis, however, shows that for delay errors of larger magnitudes, the lower limit of employable sampling rate increases in this approach. To address this, we propose a multiple delay line model where the delay values of a set of parallel delay lines are gradually increased, with each k-th delay line determining its coarse estimate by using the coarse estimate of the $(k-1)$-th delay line as reference, and fine estimation is carried out at the end from the longest delay line. The paper also discusses ways of optimizing the throughput and the number of delay lines in the multiple delay line set up.**

***Index Terms*—Frequency estimation, sub-Nyquist sampling, aliasing, delay error, electronic warfare.**

I. Introduction

Frequency estimation of radar signals [1]–[3] has remained an ever challenging task, especially in electronic support receivers (ES) [4] deployed in military aerial vehicles for electronic warfare (EW) [5], [6]. This is mainly because radar signals in ES applications span a very wide range of frequencies typically from 2 to 18 GHz, and have narrow durations (few tens of nanoseconds) in rare intervals. Digital processing [7] is preferred over analog means for this as it offers higher accuracy and better performance in the presence of time-overlapped radar signals. However, to cover such wide-band, digital methods need to employ exorbitantly high sampling rates (of the order of 36 GHz) to avoid aliasing which is practically very daunting. To overcome this, two approaches have been used conventionally. The first one of them, the so-called super-heterodyne approach, uses a narrowband bandpass filter with tunable local oscillator at the RF front end, followed by ADC and a single DSP processor. Here, the radar signal is detected by slowly sweeping the entire band (2-18 GHz). The other method, known as channelizer based approach, uses a parallel bank of bandpass filters. Like the superheterodyne receiver, each bandpass filter output is down converted to baseband which is digitized easily by existing ADCs and is then processed by DSP processor. Both these approaches, however, do not meet the requirement of modern EW receiver. Sequential nature of operation in the superheterodyne approach reduces the probability of intercept of radar signals while parallel nature of operations in channelized approach increases size, weight, power (SWaP) and cost. Operating a sub-Nyquist sampling rate and trying to estimate the frequency from phase measurements could be a viable alternative. But, this approach too faced challenges from the input bandwidth limitations of existing analog-to-digital converters (ADC). The recent emergence of wide-band track-and-hold amplifiers (wTHA) [8], [9] which can linearly track high-frequency signals (up to 18 GHz) has, however, resolved this issue to a large extent and produced a renewed interest in the domain of sub-Nyquist frequency estimation [10]–[15] for EW receivers. In the sub-Nyquist approach, frequency estimates are obtained from the phase difference between the aliased components of the original signal and a delayed version of the same with a known delay [16], [17]. One problem, however, still remains in the delay line based approach, caused by the fact that no delay, especially analog delay, is error free, and since the operating frequency range spans several GHz (which implies that the delay deployed is of a small magnitude in order to satisfy certain ambiguity constraint), even a small error in the delays employed can give rise to a significant error in the frequency estimates.[1]

In this paper, we address this issue and propose solutions to combat the effects of delay errors. Initially we propose a “two delay line” based approach where the shorter delay line produces a so-called coarse estimate of the frequency and the longer delay line treats it as a reference to resolve certain ambiguities in its coarse estimate, which is seen to be J times more accurate than the reference estimate, where J is the delay ratio between the longer and the shorter delay line. A fine estimation of the frequency is then carried out from this coarse estimate which enhances the estimation accuracy significantly. Our analysis shows that if the magnitude of the delay error is quite large, the lower limit of the employable sampling rate in the above case, however, increases, which may again raise the cost and complexities of implementation. To tackle this, we extend the above approach to a “multiple delay line” model where the delay values of a set of parallel delay lines are gradually increased, each k-th delay line determines its coarse estimate by using the coarse estimate of the $(k-1)$-th delay line as reference, and fine estimation is carried out at the end from the longest delay line. For both the coarse and fine estimation, we present throughput optimized solutions for achieving quick response time in hardware realization. Lastly, we discuss ways of optimizing the number of delay lines in a

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[1]Note that in the conventional approaches like superheterodyne receivers, as the input frequency after down conversion is much less, the delay magnitude is relatively higher and thus the delay errors do not have any tangible effect on estimation accuracy in case of delay line based, sub-Nyquist estimation. This explains why delay error effects were not considered in the conventional receivers.

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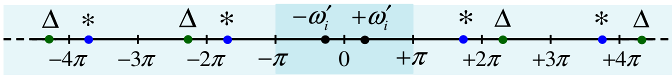

Fig. 1. Possible locations of the frequency ω_i given ω_i' ("*" and "Δ" correspond to the cases of band inversion and no band inversion respectively).

"multiple delay line" model (thus addressing the SWaP issues), assuming that some a priori knowledge of the delay error characteristics of the delay line is available. We validate the proposed methods both through MATLAB-based simulation and by developing prototype hardware.

II. Background

Consider the sinusoid $x_a(t) = A\cos(\Omega_i t + \phi)$ as received by a radar with frequency $f_i = \frac{\Omega_i}{2\pi}$ in the range of $2-18$ GHz, amplitude A and phase $\phi \in [-\pi,\ \pi]$, which is sampled at a rate $\frac{1}{T}$, generating the sequence $x(n) = A\cos(\omega_i n + \phi)$, $\omega_i = \Omega_i T$. The discrete time Fourier transform (DTFT) $X(e^{j\omega})$ of $x(n)$ (periodic in ω over 2π) consists of two impulses of strengths $\frac{\pi}{T}Ae^{j\phi}$ and $\frac{\pi}{T}Ae^{-j\phi}$ at ω_i and $-\omega_i$ respectively and their left and right shifted versions by integer multiples of 2π. If sampled at higher than the Nyquist rate (i.e., if $\Omega_i < \frac{\pi}{T}$), ω_i and $-\omega_i$ will come within $-\pi$ and π, meaning, one can directly obtain ω_i and thus Ω_i from the DTFT over $-\pi$ to π and the location of the impulses contained therein. Nyquist and above rate of sampling, however, is not practically realizable in this case, as that would mean that the signal has to be sampled at an exorbitantly high sampling rate (in the range of 36 GHz or above). On the other hand, if sampled below Nyquist rate, the above two impulses will come as alias components, placed at two new frequencies $\pm\omega_i'$ $(0 \le \omega_i' \le \pi)$. Two cases can arise here [18], namely,

- Case 1 : $2l\pi < \omega_i < (2l+1)\pi$ for some integer $l,\ l \ge 0$. In such case, the impulses with strength $\frac{\pi}{T}Ae^{j\phi}$ and $\frac{\pi}{T}Ae^{-j\phi}$ will be mapped from ω_i and $-\omega_i$ to ω_i' and $-\omega_i'$ respectively, where $\omega_i' = \omega_i - 2l\pi$ (for some integer $l,\ l \ge 0$ so that $0 \le \omega_i' \le \pi$), or, equivalently, $\omega_i = \omega_i' + 2l\pi$.
- Case 2 : $(2l+1)\pi \le \omega_i < 2(l+1)\pi$ for some integer $l,\ l \ge 0$. In such case, the impulses with strength $\frac{\pi}{T}Ae^{j\phi}$ and $\frac{\pi}{T}Ae^{-j\phi}$ will be mapped to $-\omega_i'$ and $+\omega_i'$ respectively, i.e., there will be *band inversion*, where $\omega_i' = -\omega_i + 2(l+1)\pi$ (for some integer $l,\ l \ge 0$ so that $0 \le \omega_i' \le \pi$), or, equivalently, $\omega_i = -\omega_i' + 2(l+1)\pi$.

The above is summed up in Fig. 1. Clearly, in this case, one cannot obtain ω_i from the measured ω_i', since the value of l is not known. One way to overcome this problem will be to consider a delayed version of the signal, say, $x_{a,d}(t) = A\cos(\Omega_i t - \Omega_i\tau + \phi)$, where τ is a known delay (to avoid ambiguity, it is required to constrain τ as $\Omega_i\tau < 2\pi$), and sample it at the same sub-Nyquist rate. The resulting sequence $x_d(n) = A\cos(\omega_i n - \Omega_i\tau + \phi)$ will have its DTFT $X_d(e^{j\omega})$ identical to that of $X(e^{j\omega})$, except that the strength of the impulses will change from $\frac{\pi}{T}Ae^{j\phi}$ and $\frac{\pi}{T}Ae^{-j\phi}$ to $\frac{\pi}{T}Ae^{j\phi-\Omega_i\tau}$ and $\frac{\pi}{T}Ae^{-j\phi+\Omega_i\tau}$ respectively. By measuring the phase of any one of the two impulses, before and after delay, one can then obtain $\Omega_i\tau$ and thus Ω_i. This approach is still not free of ambiguity, since it is not known whether band inversion has taken place or not (note that phase difference measured at ω_i' between the original and the delayed signals is $\Omega_i\tau$ in case of no band inversion and $-\Omega_i\tau$ in case of band inversion). To resolve this, the above ambiguity constraint is modified further as $\Omega_i\tau < \pi$. From this and the fact that $-\pi < \phi < +\pi$, as per the convention adopted, the following can be concluded :

- Under Case 1 above, the phase of the delayed signal at ω_i' will be either $\phi - \Omega_i\tau$ if $\phi - \Omega_i\tau \ge -\pi$, and $\phi - \Omega_i\tau + 2\pi$ if $\phi - \Omega_i\tau < -\pi$. Therefore, the phase difference between the original and the delayed signals at ω_i', say, ϕ_d will be either positive (i.e., $\Omega_i\tau$) in the range $[0,\ \pi]$, or, negative (i.e., $\Omega_i\tau - 2\pi$) in the range of $[-\pi,\ -2\pi]$.
- Under Case 2 above, the impulse at ω_i' will have phase opposite of that of the impulse at ω_i. Following a logic similar to above, ϕ_d will be either negative (i.e., $-\Omega_i\tau$) in the range $[0,\ -\pi]$, or positive (i.e., $-\Omega_i\tau + 2\pi$) in the range $[\pi,\ 2\pi]$.

From the range of measured ϕ_d, one can make out whether band inversion has taken place. The frequency Ω_i is then evaluated as $\Omega_i = \frac{\phi_{d,1}}{\tau}$, where $\phi_{d,1}$ is given by (a) ϕ_d if $0 < \phi_d < \pi$, (b) $2\pi - \phi_d$ if $\pi \le \phi_d < 2\pi$, (c) $-\phi_d$ if $-\pi < \phi_d \le 0$, and (d) $\phi_d + 2\pi$ if $-2\pi < \phi_d \le -\pi$.

Fine Estimation of the Frequency : The above produces the correct estimate of Ω_i in an ideal scenario having no sources of error. In practice, however, the estimation procedure is affected by various forms of errors, resulting in a coarse estimate $\hat{\Omega}_c = \Omega_i + \nu$ where ν is an estimation error. In such case, the degrading influence of ν can be overcome and almost accurate estimate of Ω_i can be obtained, by determining an integer l so that $\omega_i' + 2l\pi$ (in case of no band inversion), or, $-\omega_i' + 2(l+1)\pi$ (in case of band inversion) is nearest to the measured $\hat{\Omega}_c$. This generates the so-called fine estimate of Ω_i ($\frac{1}{T}(\omega_i' + 2l\pi)$ or $\frac{1}{T}(-\omega_i' + 2(l+1)\pi)$).

III. Proposed Methods to Combat the Effects of Delay Errors

In the above, the delay τ of the delay line was assumed to be known correctly. In practice, however, no delay line is perfect and every delay line is usually associated with some unknown delay error, say, $\Delta\tau$, producing a total delay of $\tau + \Delta\tau$. Further, $\Delta\tau$ is not constant but is dependent on frequency which is unknown and thus cannot be offset a priori. Fig. 2 shows a practical delay error measurement setup and a typical case of delay error variation over frequency in the range of $2-18$ GHz in an analog delay line based prototype hardware.

Taking such delay error into account and under the ambiguity constraint $\Omega_i(\tau + \Delta\tau) < \pi$, one obtains from the above a coarse estimate $\hat{\Omega}_c = \Omega_i + \frac{\Omega_i\Delta\tau}{\tau}$. Consequently, a fine estimate (ideally having zero error) of Ω_i, say, $\hat{\Omega}_f$, will be either $\Omega_i' + \Omega_s l$, or, $-\Omega_i' + \Omega_s(l+1)$ for an appropriate l $(l = 0, 1, 2, \cdots)$, where $\Omega_i' = \frac{\omega_i'}{T}$ and $\Omega_s = \frac{2\pi}{T}$ is the analog sampling frequency (radian per sec). In the proposed approach, we choose l from $0, 1, 2, \cdots$ so that $\Omega_i' + \Omega_s l$, or, $-\Omega_i' + \Omega_s(l+1)$ is closest to $\hat{\Omega}_c$. The corresponding $\Omega_i' + \Omega_s l$, or, $-\Omega_i' + \Omega_s(l+1)$ will then give the correct Ω_i, provided, as shown in Fig. 3(a), $\Omega_i\frac{|\Delta\tau|}{\tau} < \frac{\Omega_s}{2}$, or, equivalently,

$$\frac{\Omega_s}{\Omega_i} > \frac{2|\Delta\tau|}{\tau}. \tag{1}$$

The above gives a lower bound on the sampling frequency Ω_s as a fraction (i.e., $\frac{2|\Delta\tau|}{\tau}$) of Ω_i. For very small $\frac{|\Delta\tau|}{\tau}$, even when we take Ω_i corresponding to the highest signal frequency in the given band (i.e., 18 GHz), the above lower bound will still be sufficiently less and thus, one can choose suitable, sub-Nyquist sampling frequency Ω_s for the entire frequency range of interest. In practice, however, one often encounters larger delay errors and typical value of $\frac{|\Delta\tau|}{\tau}$ is in the range of 0.1. For such cases, taking Ω_i to be the highest in the band (i.e., $2\pi \times 18$ Giga radian/sec), we observe that $F_s = \frac{\Omega_s}{2\pi}$ should be above 3.6 GHz, which is still high for practical realizations. In the following, we show how fine frequency estimation can still be carried out in such cases.

A. Two Delay Lines

In this case, we consider three signals :

(a) The original signal $s_a(t) = A\cos(\Omega_i t + \phi)$.

(b) The delayed version of $s_a(t)$, where it is delayed by $\tau + \Delta\tau$ as discussed above ($\Omega_i(\tau + \Delta\tau) < \pi$). As before, we evaluate the phase difference ϕ_d^0 between the original and the delayed signal. One coarse estimate of Ω_i is then obtained as $\hat{\Omega}_c^0 = \frac{\phi_d^0}{\tau} = \Omega_i + \frac{\Omega_i \Delta\tau}{\tau}$.

(c) Another delayed version of $s_a(t)$ where it is delayed by $J\tau$ ($J > 1$). Let $\Delta\tau'$ be the corresponding delay error. If J is not too large, it is reasonable to assume that $|\Delta\tau'| \approx |\Delta\tau|$. As the delay is increased J times, it is expected that there will be ambiguity in phase measurement in this case, i.e., $\Omega_i(J\tau + \Delta\tau') > 2\pi$.

Again, we evaluate the phase difference, say, ϕ_d^1 between the original signal and the delayed signal in (c). Because of the above stated ambiguity, we will have $\phi_d^1 = \Omega_i(J\tau + \Delta\tau') - 2\pi m$, where m is an integer so that $0 < \phi_d^1 < 2\pi$ [2]. This will

[2]Note that since from step (b), it is known whether band inversion has taken place or not, there is no ambiguity due to this in step (c) in measuring ϕ_d^1.

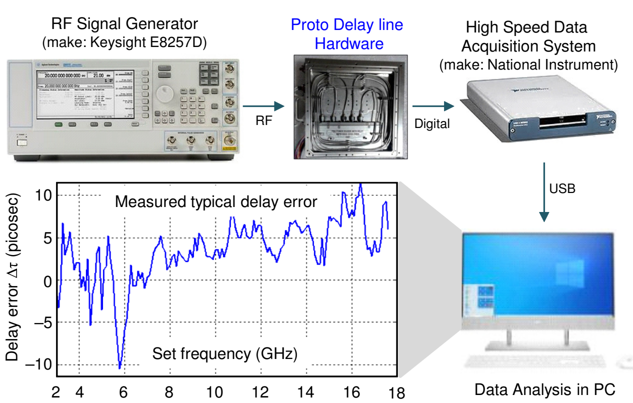

Fig. 2. Delay error variation measurement setup showing a typical case of variation over frequency in the range of 2 - 18 GHz in an analog delay line based prototype hardware.

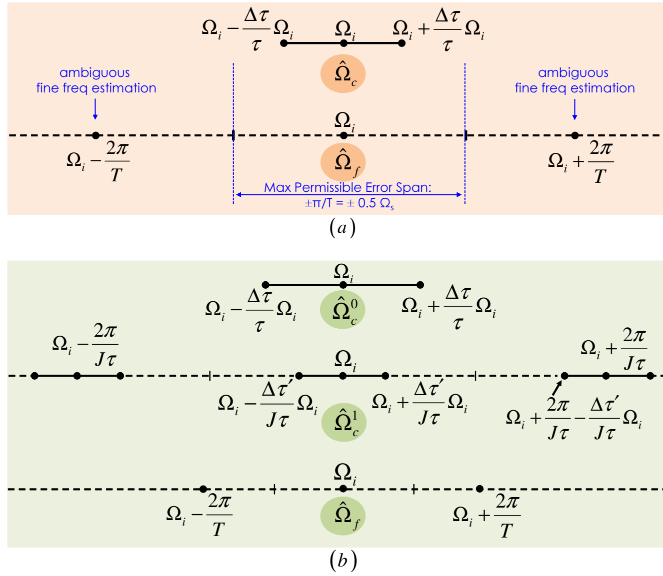

Fig. 3. Fine frequency estimation: (a) small delay error case in single delay line, (b) large delay error case in two delay lines

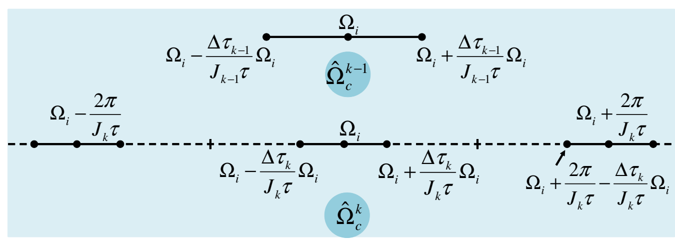

Fig. 4. Coarse frequency estimation in case of multiple delay lines

result in the following frequency estimate : $\hat{\Omega}_c^1 = \frac{\phi_d^1}{J\tau} + \frac{2\pi m}{J\tau} = \Omega_i + \frac{\Omega_i \Delta\tau'}{J\tau}$. The task now is to determine m.

In the proposed approach, we choose that value of m for which $\hat{\Omega}_c^1$ is closest to $\hat{\Omega}_c^0$. For this to produce the correct value of m, as seen from Fig. 3(b), the following condition should be satisfied as $\frac{2\pi}{J\tau} - |\hat{\Omega}_c^0 - \hat{\Omega}_c^1| > |\hat{\Omega}_c^0 - \hat{\Omega}_c^1|$. As $|\Delta\tau'| \approx |\Delta\tau|$, for deriving a sufficient condition, we take the case : $\Delta\tau' = -\Delta\tau$. Substituting the expressions of $\hat{\Omega}_c^1$ and $\hat{\Omega}_c^0$ in the above expression, after some simplifications, we arrive at the following condition :

$$\Omega_i|\Delta\tau| \;<\; \frac{\pi}{J+1}. \tag{2}$$

Note that once the correct m is obtained, we have $\hat{\Omega}_c^1 = \Omega_i + \frac{\Omega_i \Delta\tau'}{J\tau}$, meaning, due to division by J, the magnitude of the error component is reduced by approximately J times as compared to earlier. We now carry out fine estimation of Ω_i using signals in (a) and (c). It is easy to see that the condition for correct measurement of Ω_i in this case (i.e., analog of (1)) will be given by,

$$\frac{\Omega_s}{\Omega_i} > \frac{2|\Delta\tau|}{J\tau}. \tag{3}$$

Due to division by J on the right hand side, in this case, the lower bound on the sampling frequency will be much less and thus, practically employable.

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B. Multiple Delay Lines

From (2), it is seen that for large delay error, J should be small to satisfy (2). However, from (3), it is seen that smaller J results in larger sampling frequency which is often not practical. In such cases, in addition to the original delay line of delay τ, we employ, say, q additional parallel delay lines with delays $J_1\tau,\ J_2\tau, \cdots,\ J_q\tau$ where $1 = J_0 < J_1 < J_2 < \cdots < J_q$. Of these additional parallel, the first $(q-1)$ carry out only coarse estimation while the q-th delay line carries out both coarse as well as fine estimation of the frequency. Thus, the condition in (3) does not apply to these first $(q-1)$ additional delay lines. Also, we start with a small value of J_1 (preferably close to one) and progressively move upwards. Then, as above, the phase difference, say, ϕ_d^k between the original signal and the delayed signal from the k-th delay line, $k = 1, 2, \cdots, q$, will be given by $\phi_d^k = \Omega_i(J_k\tau + \Delta\tau_k) - 2\pi m$, where m is an integer which is to be determined so that $0 < \phi_d^k < 2\pi$ and $\Delta\tau_k$ is the delay error associated with the k-th delay line. As before, this will result in the following coarse estimate : $\hat{\Omega}_c^k = \frac{\phi_d^k}{J_k\tau} + \frac{2\pi m}{J_k\tau} = \Omega_i + \frac{\Omega_i\Delta\tau_k}{J_k\tau}$. However, unlike above, we will now choose that value of m for which $\hat{\Omega}_c^k$ is closest to $\hat{\Omega}_c^{k-1}$ This is shown in Fig. 4. Clearly, this requires $\frac{2\pi}{J_k\tau} - \left|\hat{\Omega}_c^{k-1} - \hat{\Omega}_c^k\right| > \left|\hat{\Omega}_c^{k-1} - \hat{\Omega}_c^k\right|$. As before, assuming that $|\Delta\tau_k| \approx |\Delta\tau|,\ k = 1, 2, \cdots, q$, substituting the expressions of $\hat{\Omega}_c^{k-1}$ and $\hat{\Omega}_c^k$, we obtain, after some simplifications,

$$\Omega_i\left|\Delta\tau\right| \ < \ \frac{\pi}{1 + \frac{J_k}{J_{k-1}}}. \tag{4}$$

Since we maintain $\frac{J_k}{J_{k-1}}$ relatively small for all the delay lines, the above allows coarse estimation in presence of reasonably large delay errors in each line. At the same time, as the delay increases progressively from $J_1\tau$, we arrive at a sufficiently large $J_q\tau$ so that as fine frequency estimation is carried out by the q-th delay line, the condition $\frac{\Omega_s}{\Omega_i} > \frac{2|\Delta\tau|}{J_q\tau}$ (which is the analog of (3)) results in much less lower limit of the sampling frequency as is desirable[3].

C. High Throughput Implementation

As discussed in the multiple delay lines case, the integer values m need to be known at stages to resolve ambiguities in $\hat{\Omega}_c^k$ in a progressive way from J_{k-1} to J_k for every value of $k = 2, 3, \cdots, q$. This requires search operations to find out which $\hat{\Omega}_c^k$ is closest to $\hat{\Omega}_c^{k-1}$ and is obviously a computationally expensive as well as low throughput solution towards hardware implementation. To alleviate the same, we propose a low complexity high-throughput look-up-table (LUT) based solution. Let us assume that $\hat{\Omega}_c^{k-1}$ is already estimated as $\hat{\Omega}_c^{k-1} = \Omega_i + \frac{\Omega_i\Delta\tau_{k-1}}{J_{k-1}\tau}$ and further, $\hat{\Omega}_c^k$ is to be estimated once m is determined. We define $\hat{\Omega}_c^{k'} = \frac{\phi_d^k}{J_k\tau}$. Then, from above, we have, $\hat{\Omega}_c^{k'} = \hat{\Omega}_c^k - \frac{2\pi m}{J_k\tau} = \Omega_i - \frac{2\pi m}{J_k\tau} + \frac{\Omega_i\Delta\tau_k}{J_k\tau}$. From these two expressions of $\hat{\Omega}_c^{k-1}$ and $\hat{\Omega}_c^{k'}$, m can then be written as, $m = M + \Delta M$ where $M = \frac{J_k\tau}{2\pi}\left(\hat{\Omega}_c^{k-1} - \hat{\Omega}_c^{k'}\right)$ and $\Delta M = \frac{\Omega_i}{2\pi}\left(\Delta\tau_k - \Delta\tau_{k-1}\cdot\frac{J_k}{J_{k-1}}\right)$. From (4), it is easy to

[3]Note that this gain is, however, achieved at the cost of more hardware and power as several delay lines are deployed in parallel.

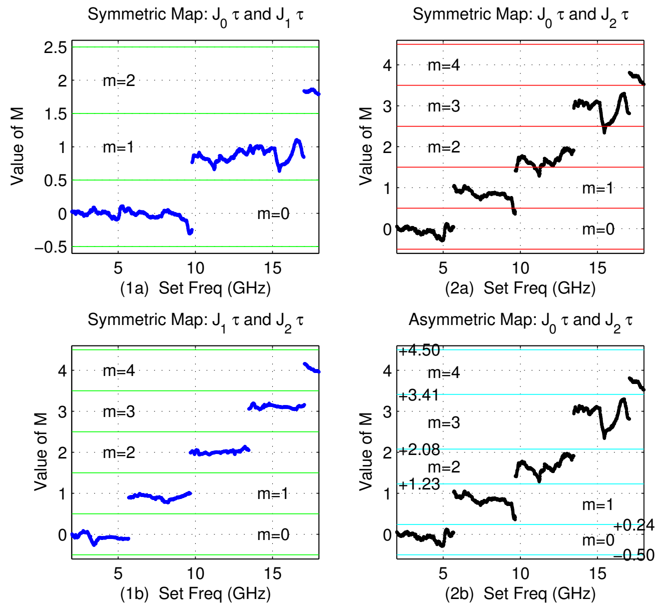

Fig. 5. (1) Results in 3-DL case showing values of M not crossing its symmetric limits $\pm\frac{1}{2}$ around m for $J_0\tau$, $J_1\tau$ pair (1a) and for $J_1\tau$, $J_2\tau$ pair (1b); (2) Results in 2-DL case showing values of M crossing its symmetric limits $\pm\frac{1}{2}$ around m for $J_0\tau$, $J_2\tau$ pair (2a) but not crossing its asymmetric limits for $J_0\tau$, $J_2\tau$ pair (2b).

check that $\Delta M \leqslant \pm\frac{1}{2}$. From this and the fact that m is an integer, we can write,

$$m = NINT(M), \tag{5}$$

where $NINT(x)$ equals to the integer nearest to x. Clearly, M has equal boundaries of 0.5 on either side of the integer m which we refer as symmetric boundaries in the paper. So, once M is computed as $\frac{J_k\tau}{2\pi}\left(\hat{\Omega}_c^{k-1} - \hat{\Omega}_c^{k'}\right)$ using subtraction and constant factor multiplication operations, (5) can easily be implemented using symmetric LUT as follows: $m = 0$ for $M \leqslant 0.5$, $m = 1$ for $0.5 < M \leqslant 1.5$, $m = 2$ for $1.5 < M \leqslant 2.5$ and so on.

The algorithmic flow chart of the proposed sub-Nyquist frequency estimation scheme in a multiple delay line set up is given in Table 1.

IV. Simulation and Implementation Results

Here we use physical co-axial cables to generate pico-second time-delay to cover such wide-band. To cover entire $2-18$ GHz with guard band on either side, we choose $\tau = 27$ ps time delay using co-axial cable with dielectric constant $\epsilon_r = 2.2$ and length of $l = \frac{c\tau}{\sqrt{\epsilon_r}} = 5.46$ mm. One proto-hardware, as shown in Fig. 2, has been developed to study the practical delay errors and develop a model for the same to use in simulation. From this proto-hardware, maximum $\Omega_i|\Delta\tau|$ over full frequency band is found to be $52°$. As per (1), this demands the sampling frequency to be higher than 10.7 GHz which is practically infeasible. This prompts us to use multiple delay lines (DLs) with delays, say, $J_1\tau,\ J_2\tau, \cdots,\ J_q\tau$, as presented in Section III.B. Replacing J by J_q in (3) and

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Table 1 : Sub-Nyquist frequency estimation in the multiple delay line setup towards ES receiver design.

A. Setting up the system

1. Fix the *shortest delay* line l with equivalent delay τ such that $l = \frac{c\tau}{\sqrt{\epsilon_r}}$ where $\tau = \frac{\pi}{\Omega_i}$ and Ω_i can be treated as maximum frequency of operation.
2. Fix the *sampling frequency* as Ω_s that is practically feasible.
3. In laboratory, measure the quantity $\Omega_i\big|\Delta\tau\big|$ by varying Ω_i over the operating frequency range and find its maximum value.
4. Find the minimum value of J_q from the relation (Eq.(3) in paper): $J_q > 2\,\frac{\Omega_i|\Delta\tau|}{\Omega_s\tau}$ to constitute the *longest delay* $J_q\tau$.
5. Taking $\frac{J_k}{J_{k-1}}$ (say, r) to be equi-ratio for all k, find out the maximum value of r from the relation (Eq.(4) in paper): $\frac{J_k}{J_{k-1}} < \frac{\pi}{\Omega_i|\Delta\tau|} - 1$.
6. Fix r and integer value of q such that $r^q = J_q$ to constitute the $(q-1)$ *intermediate delays*, i.e., $J_1\tau, \cdots, J_k\tau, \cdots, J_{q-1}\tau$.

B. Frequency estimation

- Initialization: Measure ϕ_d^0 and calculate $\hat{\Omega}_c^0 = \frac{\phi_d^0}{J_0\tau}$.
- **for** $k = 1, 2, \cdots, q,$
  - Measure ϕ_d^k, i.e., phase difference between the original and the delayed signal from the k-th delay line ($0 \leq \phi_d^k < 2\pi$)
  - Calculate $\hat{\Omega}_c^{k'}$ using $\hat{\Omega}_c^{k'} = \phi_d^k/(J_k\tau)$.
  - Calculate $m = \text{NINT}(M)$, where $M = \frac{J_k\tau}{2\pi}\big(\hat{\Omega}_c^{k-1} - \hat{\Omega}_c^{k'}\big)$.
  - Update coarse frequency estimate as $\hat{\Omega}_c^k = \hat{\Omega}_c^{k'} + \frac{2\pi m}{J_k\tau}$.

  end
- Denote final *coarse frequency estimate* as $\hat{\Omega}_c^q$.
- Measure Ω_i' which is the frequency location of the observed aliased component in the range : $\big[-\Omega_s/2, \Omega_s/2\big]$.
- **Output:** Calculate *fine frequency estimate* as

$$\hat{\Omega}_f = \begin{cases} \Omega_i' + \Omega_s l, & \text{no band inversion} \\ -\Omega_i' + \Omega_s(l+1), & \text{band inversion} \end{cases}$$

where

$$l = \begin{cases} \text{NINT}\left[\frac{\hat{\Omega}_c^q - \Omega_i'}{\Omega_s}\right], & \text{no band inversion} \\ \\ \text{NINT}\left[\frac{\hat{\Omega}_c^q + \Omega_i'}{\Omega_s}\right] - 1. & \text{band inversion} \end{cases}.$$

also making the following substitutions : $\Omega_i|\Delta\tau| = 52^\circ$, $\Omega_s = 2\pi \times 2500 \times 10^6$ radian/sec, and $\tau = 27 \times 10^{-12}$ sec, we find that $J_q \geq 4.28$. Taking $\frac{J_k}{J_{k-1}}$ to be same for all k, we get minimum value of q to be 2, since, in such case, we have, $\frac{J_2}{J_1} = \frac{J_1}{J_0} = 2.07$ (i.e., $\sqrt{4.28}$) which leads to the satisfaction of (4) with $\Omega_i|\Delta\tau| = 52^\circ$. We consider sinusoidal radar signal and vary frequency between 2 to 18 GHz in steps of 10 MHz. The results as shown in Fig. 5-(1a) & 5-(1b) show that the corresponding values of M are not crossing their symmetric boundary limits of $\pm\frac{1}{2}$ around m, and thus, values of m for (J_0, J_1) and (J_1, J_2) can be obtained without any error to find the coarse estimate unambiguously.

Optimizing the Number of Delay Lines: Here, instead of three, we take two DLs of delay, say, $J_0\tau$ and $J_2\tau$ which are in $1 : 4.28$ ratio and obtain values of M as shown in Fig. 5 (2a). Clearly and as expected, the values of M are exceeding their symmetric boundary limits of $\pm\frac{1}{2}$ as $\Omega_c|\Delta\tau| = 52^\circ$ exceeds its limit $\frac{\pi}{1+4.28}$ rad, or 34° from (4). This means, the values of m cannot be derived correctly. However, it is also evident that by changing the symmetric boundary values to asymmetric ones, we can determine m correctly. Such re-assignment of boundary values is possible as the pre-computed practical data is available a priori from the laboratory experiments. This is shown in Fig. 5 (2b). Here, the asymmetric boundary values are chosen such that they provide maximum gap or distance from extreme values in the corrsponding cluster on either side. This proposed scheme, termed as *"asymmetric map resolver"*, is advantageous provided there is no overlap between the clusters or zones associated with different values of m. As seen above, this can reduce the requirement of number of DLs and their associated hardware (e.g., in this case, $J_1\tau$ DL is not required leading to about 30 % saving in SWaP).

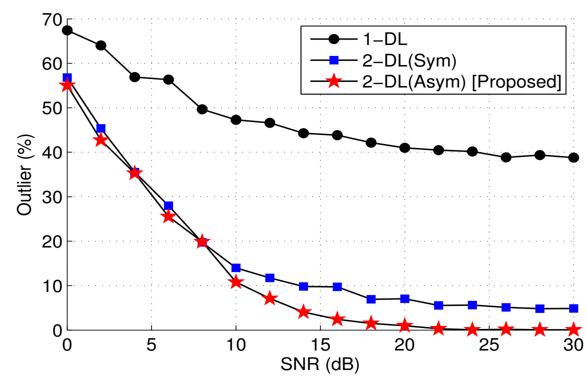

Fig. 6. Performance Gain Comparison showing Outlier(%) among 1-DL[17], 2-DL (Sym Map) and 2-DL (Asym Map) under varying SNR for typical large delay error case ($\big|\frac{\Delta\tau}{\tau}\big| > 10\%$).

Next, we carry out MATLAB based simulation and show performance comparison among the 1 DL ($J_0\tau$) [17], 2 DLs ($J_0\tau$ and $J_2\tau$) with "symmetric map" and same 2 DLs with proposed "asymmetric map" cases under varying SNR. The performance metric is taken as "outlier (%)" which is defined as the number of large frequency errors (normalized to 100) that occur due to incorrect m (caused by delay errors) in estimating the frequency [4]. The simulation results are shown in Fig. 6 which confirm that the proposed multiple DLs with "asymmetric map" outperforms not only the 1 DL

[4]We have seen that correct determination of m requires satisfaction of the condition $\Delta M \leqslant \pm\frac{1}{2}$. If not satisfied, it will lead to wrong determination of m and thus, the wrong estimation of the final coarse frequency $\hat{\Omega}_c^q$ which, in turn, will lead to wrong calculation of l. For every increase or decrease of l, the fine frequency estimate jumps by multiple of Ω_s, producing large estimation error.

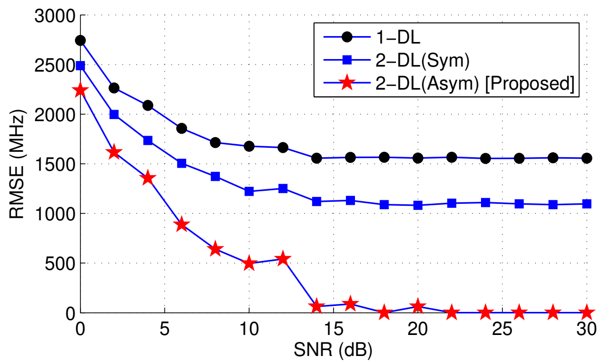

Fig. 7. Performance Gain Comparison showing Root Mean Square Frequency Error (RMSE) among 1-DL[17], 2-DL (Sym Map) and 2-DL (Asym Map) under varying SNR for typical large delay error case ($\left|\frac{\Delta\tau}{\tau}\right| > 10\%$).

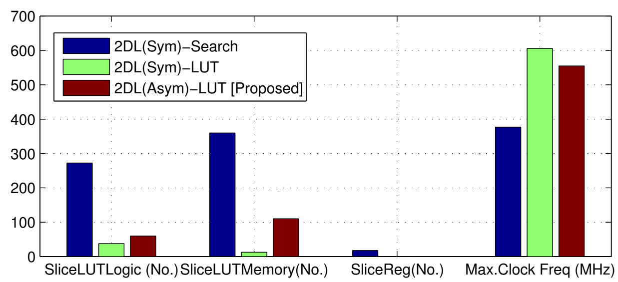

Fig. 8. FPGA resource utilization and maximum achievable clock frequency figures for comparison among search based and mapping based (symmetric and asymmetric) LUT schemes.

but even the 2 DLs with "symmetric map", thus providing better performance under varying SNR condition. Fig. 7 also displays frequency estimation accuracy by plotting the root mean square error (RMSE) over varying SNR, which confirm the same. Lastly, we also coded the proposed "LUT-based implementation" scheme for 2 DLs with "asymmetric map" along with other schemes (e.g., 2 DLs with "search" and "symmetric map") in VHDL, targeting AMD-Xilinx Artix-7 series FPGA (Part No. XC7A200T-FFG1156-2). Synthesis, mapping and place & routing were carried out through AMD-Xilinx Vivado 2016.4 IDE. Fig. 8 shows that the proposed "LUT-based implementation" scheme also outperforms search-based scheme in terms of FPGA resource and achievable maximum clock frequency (i.e. throughput). However, the proposed "asymmetric map" technique which optimises and reduces overall number of DLs demands subtle increase in FPGA resources and reduction in throughput as compared to same number of DLs with "symmetric map".

V. Conclusion

We have proposed methods to achieve robustness against delay errors in delay line based sub-Nyquist frequency estimation over the $2-18$ GHz band. The delay τ employed in the delay line for such high frequency range is usually small to maintain the ambiguity constraint $\Omega_i\tau < \pi$. This implies that the relative delay errors (i.e., $\left|\frac{\Delta\tau}{\tau}\right|$) usually have large magnitude and in such cases, the existing single delay line based solution is likely to result in large outliers in the frequency estimation leading to poor estimation accuracy. To alleviate the same, we first presented a two delay line based method which carries out coarse estimation followed by fine estimation of the frequency. This is then extended to a multiple delay line based scheme to handle delay errors that are even larger. A throughput optimized solution for hardware realization of the multiple delay line based set up is presented next. We also discussed ways of optimizing the number of delay lines being used. The proposed algorithms were validated through extensive MATLAB based simulation studies using practical data obtained from prototype delay line based hardware.

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